Power Optimal Routing in Wireless Networks

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***Abstract*— Reducing power consumption and increasing battery life of nodes in an ad-hoc network requires an integrated power control and routing strategy. Power optimal routing selects the multi-hop links that require the minimum total power cost for data transmission under a constraint on the link quality. This paper studies optimal power routing under the constraint of a fixed end-to-end probability of error and compares the power optimal routes obtained with this criterion with those from the more commonly used fixed per hop error rate constraint. The comparison is carried out by looking at the properties of the power optimal graph, formed by the union of all the power optimal routes. The paper also provides algorithms to determine the power optimal routes.**

***Index Terms*— Sensor Networks, Power Control, Routing.**

I. Introduction

In multi-hop networks link reliability, availability, delay and, last but not least, the battery life of each node are all entangled through a unique variable, the power spent on each bit transferred. Power control issues have been addressed for quite some time in the literature, especially in the context of cellular networks [12]. While the dependence between multiple access control and power control is also evident in cellular networks, the trade-mark of multi-hop networks is the interdependence between routing and power control. In [6] the optimal transmission radius in multi-hop wireless networks was derived under the constraint that all nodes transmit the same power, which was later relaxed in [7]. In addition to draining the battery of the node, since a wireless link is a broadcast mechanism, increasing the power used to transmit a packet might cause other side-effects such as interference with other nodes in the network. Therefore, it is important to determine the minimum power necessary to route a packet, and recent work in ad-hoc network has focused on the problem of optimized routing that minimize the total path power consumption, see e.g. [11], [15], [10].

Given the per hop transmit power for a packet and its route, the power cost of a path is defined to be the sum of the per-hop transmit power along the path. Routes that minimize the power cost under a quality of service constraint are said to be *power optimal*. The union of the optimal multi-hop paths between all pairs of nodes for a specific optimality criterion form a graph, which we will refer to as the *power optimal graph*. The power optimal graph is the ensemble of optimum physical (single and multi-hop) links supporting peer-to-peer transmission for each pair of nodes optimally. Clearly the power cost of each route depends on the optimality criterion chosen and a natural question that arises is how the optimal graph behaves as a function of the optimality criterion. A study of the properties of the power-optimum graph for a fixed per hop error rate is in [1] where it was shown that the power optimal graph with a fixed per hop error rate constraint has no crossing edges. As will be clarified in Section II, the problem with using a per hop error rate constraint is that the quality of the end-to-end connection is not guaranteed. As a result, paths with larger number of hops will produce not only an increased delay but also an increased error rate.

This paper derives a solution for the power optimum route problem under an end-to-end error rate constraint and carries a comparative study of the power optimal graphs obtained with those obtained with a per hop error rate constraint. Note that we do not consider multiple access issues as we assume an ideal scenario where all transmissions are scheduled to occur at different times (or over-different bands simultaneously), similarly to [4]. The symbol error rate expression is calculated accordingly, without considering the effect of multi-access interference. We also do not investigate delay constraints.

The interesting conclusion of our study is that the end-to-end fixed error rate power optimal graph always contains the per-hop fixed error rate power optimal graph, with some additional edges which asymptotically, as the end-to-end error rate required $\epsilon \to 0$, tend to vanish.

The paper is organized as follows: in Section II we introduce the power optimal routing with a constant end-to-end constraint, providing general bounds that allow to simplify the solution of the problem for arbitrary error rate (SER) expressions. Specific SER models are introduced in Section III. The properties of the power optimal graph are studied in Section IV and in Section V we provide a distributed algorithm to calculate the power optimal paths. *All the proofs for the Theorems and Lemmas in the following can be found in [16] and will be omitted for brevity.*

II. Constant End-to-End Error Rate

We consider the case when a packet is transmitted from a source to its destination along multiple hops where there is some probability of error per hop that depends on the distance between the hops and the transmit power. In existing work, the assumption made is that if the received power (signal-to-noise ratio) $P_{\rm recv} \geq \gamma$ where γ is some constant, then the packet is successfully received; otherwise the packet is lost. The symbol error rate (SER) is a monotonically decreasing function of $P_{\rm recv}$ therefore, for each hop $P_{\rm recv}$ can be made large enough that the SER for the hop satisfies

$$SER \leq SER(\gamma) \tag{1}$$

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Since any transmission will use the minimum amount of power required to meet the necessary error rate, this means that $SER = SER(\gamma)$. Assuming that errors per hop are independent, this implies that the end-to-end error rate SER_{e2e} is given by

$$SER_{e2e} = 1 - (1 - SER(\gamma))^N \quad (2)$$

where N is the number of hops along the path. SER_{e2e} is monotonically increasing with N.

Instead of using a routing scheme and power cost metric that allows the end-to-end error rate to increase with hop count, we examine the effect of constraining the end-to-end error rate to be a constant. In the following section, we formulate this optimization problem.

A. Optimization Problem

Let $\mathbf{X} = (X_0, X_1, \ldots, X_N)$ be an N-hop path from node X_0 to X_N that passes through nodes X_1, X_2, …, X_{N-1} in that order. Let P_i be the power allocated for the hop (X_{i-1}, X_i), and let SER_i be the corresponding symbol error rate for the hop $(i = 1, 2, \ldots, N)$. Assuming independent errors per hop, the end-to-end SER is given by

$$SER_{e2e} = 1 - \prod_{i=1}^{N}(1 - SER_i) \quad (3)$$

We define the power cost function as follows:

$$\mathcal{PC}^*(\epsilon; \mathbf{X}) = \min_{P_1,\ldots,P_N} \sum_{i=1}^{N} P_i \quad \text{given} \quad SER_{e2e} \le \epsilon \quad (4)$$

We will assume that we are interested in the case when the quantity ϵ is much smaller than one, i.e., when there is a very low probability of error.

A quick analysis shows that the optimization problem (4) leads to equations that do not have simple closed form solutions in terms of the power per hop and the power cost, even for the simplest models for the SER.

We formulate our approximate power cost metric by the following equations:

$$\mathcal{PC}(\epsilon; \mathbf{X}) = \min_{P_1,\ldots,P_N} \sum_{i=1}^{N} P_i \quad \text{given} \quad \sum_{i=1}^{N} SER_i \le \epsilon \quad (5)$$

We can justify this approximation by the following result:

Theorem 1: Assuming $\epsilon \ge 0$ and $\epsilon + 4\epsilon^2 < 1/\sqrt{2}$, and given the power cost metrics $\mathcal{PC}^*$ and $\mathcal{PC}$ defined by equations (4) and (5) respectively, we have:

$$\mathcal{PC}(\epsilon + 4\epsilon^2; \mathbf{X}) \le \mathcal{PC}^*(\epsilon; \mathbf{X}) \le \mathcal{PC}(\epsilon; \mathbf{X})$$

The proof of Theorem 1 only relies on the power cost metric being a minimization problem where the error rate constraint is met with equality at the solution. In particular, it does not depend on the expression for SER_i, or the fact that metric being minimized is the sum of the power per hop.

For the rest of this paper, we use $\mathcal{PC}$ as our power cost metric because it will lead to analytical expressions for the power cost of a path.While this is an approximation, Theorem 1 provides a way of bounding the error $\mathcal{E}(\epsilon; \mathbf{X}) = \mathcal{PC}(\epsilon; \mathbf{X}) - \mathcal{PC}^*(\epsilon; \mathbf{X})$ since

$$\mathcal{E}(\epsilon; \mathbf{X}) \le \mathcal{PC}(\epsilon; \mathbf{X}) - \mathcal{PC}(\epsilon + 4\epsilon^2; \mathbf{X}) \quad (6)$$

When we compute the power cost with specific models for SER, we will use this expression to show that the approximation error is $\mathcal{O}(\epsilon)$, and thus small compared to the power cost of the path.

Error correction mechanisms (both ARQ and FEC) can be easily included in our framework with minor changes as discussed in [16].

III. Error Rate Models

To study the optimal power allocation strategy for a path, we examine two different models for SER. Using these models, we derive analytical expressions for $\mathcal{PC}(\epsilon; \mathbf{X})$. These closed form expressions will be used in Section V to derive algorithms for computing power optimal paths.

A. Time-invariant Attenuation

The first model is a deterministic power model where we assume that the receive power of a link is attenuated by a time-invariant quantity. This attenuation coefficient can be given a physical interpretation by setting it to d^α/a, where d is the distance between the transmitter and receiver, and $\alpha > 2$ and a are constants. We can assume that the expression for SER of a link is given by:

$$SER_i = be^{-P_i/a_i} \quad (7)$$

where P_i is the transmit power and a_i is the time-invariant attenuation coefficient of the link. Equation (7) is sufficiently parameterized to be able to provide a bound for the probability of error for most digital modulations that are detected optimally in the presence of additive Gaussian noise [12].

Theorem 2: Under the assumptions of Theorem 1 and the link SER assumption given by equation (7), the optimal power cost $\mathcal{PC}(\epsilon; \mathbf{X})$ for path $\mathbf{X} = (X_0, X_1, \ldots, X_n)$ is obtained when

1) $SER_j = \epsilon \frac{a_j}{\sum_{i=1}^{N} a_i}$
2) $P_j = a_j \left(\log(b/\epsilon) + \log(\sum_{i=1}^{N} a_i/a_j)\right)$
3) $\mathcal{E}(\epsilon; \mathbf{X}) \le \log(1 + 4\epsilon) \sum_{j=1}^{N} a_j$

where a_j, SER_j, and P_j are the attenuation coefficient, link error rate, and link power allocation respectively for link (X_{j-1}, X_j).

B. Large and Small-Scale Fading

In a wireless mobile scenario the received power is affected by many more factors than the mere distance and it is common to represent the received power as a doubly stochastic random variable, with long term and short term variations [12]. The transmit power is attenuated by two factors: $G_L(t)$ caused by large-scale fading, and $G_S(t)$ caused by small-scale fading. To account for the effects of large and small-scale fading, the time-varying SER of a link is given by the random process

$$SER_i = be^{-a\Omega_{\tau_i}} \quad (8)$$

where a and b are constants, and Ω_{τ_i} is the received power, whose statistics are function of position and time. As is standard, we assume a log-normal distribution for the large-scale fading coefficient. For the small scale fading several distributions have been introduced [13] and using (8) the corresponding average SER for a given large scale fading parameter is the characteristic function of the small scale fading density. For example, for the Nakagami m-distribution the expected SER_i for a link is given by [13]:

$$SER_i = b(1 + P_i/a_i)^{-m} \tag{9}$$

where a_i is the contribution from the slow-varying large-scale fading coefficient and P_i is the average transmit power.

The Nakagami m-distribution captures the intermediate ground between strong line-of-sight and non–line-of-sight systems. Note, in fact, that Rayleigh fading is a special case of Nakagami fading when $m = 1$ while the deterministic case is obtained as $m \to \infty$.

It is not difficult to generalize our power cost expressions and power optimal routing to these scenarios:

Theorem 3: Under the assumptions of Theorem 1 and the link SER assumption given by equation (9), the optimal power cost $\mathcal{PC}(\epsilon; \mathbf{X})$ for path $\mathbf{X} = (X_0, X_1, \ldots, X_n)$ is obtained when

1) $SER_j = \epsilon \dfrac{a_j^{m/(m+1)}}{\sum_{i=1}^N a_i^{m/(m+1)}}$
2) $P_j = a_j \left((b/\epsilon)^{1/m} \left(\sum_{i=1}^N (a_i/a_j)^{m/(m+1)} \right)^{1/m} - 1 \right)$
3) $\mathcal{E}(\epsilon; \mathbf{X}) \leq \frac{4\epsilon}{m} (b/\epsilon)^{1/m} \left(\sum_{i=1}^N a_i^{m/(m+1)} \right)^{(m+1)/m}$

where a_j, SER_j, and P_j are the large-scale attenuation coefficient, link error rate, and link power allocation respectively for link (X_{j-1}, X_j).

IV. Properties of Power Optimal Paths

In this section we discuss some of the consequences of adopting an end-to-end SER constraint for power optimization. In particular, we compare our results to those reported by [1], which assumes that the amount of power required for a link (X_{i-1}, X_i) is given by

$$P_i = \log(b/\epsilon) \frac{d_i^\alpha}{a} \tag{10}$$

where a and $\alpha > 2$ are constants, and $d_i = |X_{i-1} - X_i|$ is the distance between points X_{i-1} and X_i. This model assumes that the link SER is constant ($= \epsilon$), and the attenuation coefficient a_i is given by d_i^α / a.

Under this model, the power cost of using a link does not depend on the path under consideration, unlike the link power allocation in Theorems 2 and 3. The power cost using this model, which we denote $\mathcal{KC}(\epsilon; \mathbf{X})$, is given by

$$\mathcal{KC}(\epsilon; \mathbf{X}) = \log(b/\epsilon) \sum_{i=1}^N \frac{d_i^\alpha}{a} = \log(b/\epsilon) \sum_{i=1}^N a_i \tag{11}$$

where $d_i = |X_{i-1} - X_i|$ is the distance between points X_{i-1} and X_i. Note that since ϵ only appears in the factor in front of this particular power cost metric, the best path to route a packet according to $\mathcal{KC}$ will not depend on ϵ [1].

We compare the properties of power optimal paths obtained by the metric introduced in Theorem 2 with those obtained using the metric from equation (11).

A. Comparing Power Cost Metrics

If we use equation (10) to determine the power for a hop, then the SER per hop can be as high as ϵ, thereby increasing the total end-to-end SER. The metric $\mathcal{KC}$ will therefore underestimate the amount of power required to transmit a packet along a path with an error that does not exceed ϵ. Examining the expressions in Theorem 2, we conclude that

$$\begin{aligned} \mathcal{PC}(\epsilon; \mathbf{X}) &= \sum_{j=1}^N a_j \left(\log(b/\epsilon) + \log(\sum_{i=1}^N a_i/a_j) \right) && (12) \\ &\geq \mathcal{KC}(\epsilon; \mathbf{X}) && (13) \end{aligned}$$

with equality holding if and only if we are considering a one-hop path. If we treat $p_j = a_j / \sum_{i=1}^N a_i$ as a probability distribution, observe that

$$\begin{aligned} \mathcal{PC}(\epsilon; \mathbf{X}) &= \left(\sum_{i=1}^N a_i \right) \left(\log(b/\epsilon) + \sum_{i=1}^N p_i \log 1/p_i \right) \\ &\leq \mathcal{KC}(\epsilon; \mathbf{X}) \left(1 + \frac{\log N}{\log(b/\epsilon)} \right) && (14) \end{aligned}$$

with equality holding when $p_1 = p_2 = \cdots = p_N$. When examining long paths with equidistant hops, using a simple additive cost function will underestimate the power cost by a factor that grows logarithmically with the number of hops compared to a metric that keeps the end-to-end error bounded.

B. Comparing Optimal Paths

The purpose of computing the power cost metric is two-fold. First, given a path, it determines the amount of power required to transmit a packet along that path as well as the amount of power necessary per hop. Secondly, the power cost metric allows us to compare two paths between a source and destination to determine which path would require less power for transmission. Section IV-A showed that the $\mathcal{KC}$ and $\mathcal{PC}$ metrics might differ substantially when determining the power necessary to transmit a packet along a given path. In this section, we will show that the $\mathcal{KC}$ and $\mathcal{PC}$ metrics might select different paths.

We can demonstrate that $\mathcal{PC}$-optimal paths can differ from $\mathcal{KC}$-optimal paths by means of a simple example. Consider the scenario in Figure 1 with three points A, B, and C. Without loss of generality, we can fix the distance between A and C to be one unit. Let d_1 be the distance between A and B, and let d_2 be the distance between B and C, as shown in Figure 1.

We consider optimal paths between points A and C. We can calculate the boundary between the regions where a one-hop path (A, C) is optimal and where a two-hop path (A, B, C) is optimal as a function of d_1 and d_2 according the $\mathcal{KC}$ and the $\mathcal{PC}$ metrics. Obviously d_1 and d_2 are constrained by the triangle inequality $d_1 + d_2 \geq 1$. Also, if $d_1 \geq 1$ or $d_2 \geq 1$,

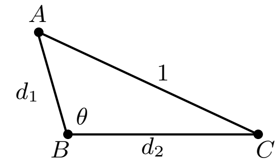

Fig. 1. Comparing paths in a three node network

then it is clear that a one-hop path is optimal for both $\mathcal{PC}$ and $\mathcal{KC}$. Therefore we restrict our attention to the region specified by the constraints $0 \leq d_1 \leq 1$, $0 \leq d_2 \leq 1$, and $d_1 + d_2 \geq 1$. These regions are shown in Figure 2, where the horizontal and vertical axes are d_1^α and d_2^α respectively, with $\alpha = 2$ and $b = 0.5$.

The points below curve **F** do not satisfy the triangle inequality and are therefore infeasible. All the other lines show the boundary between a one-hop path and a two-hop path according to different power cost metrics. Curve **L** corresponds to $\mathcal{PC}(\epsilon + 4\epsilon^2; \cdot)$ and curve **U** corresponds to $\mathcal{PC}(\epsilon; \cdot)$, for $\epsilon = 3 \times 10^{-2}$. The true boundary for $\mathcal{PC}^*$ is between these two lines by Theorem 1. Curve **K** corresponds to $\mathcal{KC}(\epsilon; \cdot)$. This plot shows that for all points between curves **U** and **K**, the optimal path selected based on $\mathcal{KC}$ differs from the optimal selected based on $\mathcal{PC}^*$.

C. Dependence of Paths on Error Rate

As noted above, the optimal routes in terms of power cost according to metric $\mathcal{KC}$ will not depend on the value of ϵ. In this section we show that $\mathcal{PC}$-optimal paths change if we change ϵ.

Consider the example shown in Figure 1 with the same parameters as in Section IV-B. Figure 3 is similar to Figure 2, except we show the boundary between one-hop and two-hop paths for different values of ϵ. It is evident that the region between curves **U** and **L2** corresponds to cases where a one-hop path is optimal for $\epsilon = 3 \times 10^{-2}$, whereas a two-hop path is optimal if $\epsilon = 10^{-3}$.

Examining the equation for the power cost metric in (12) the term $\log(b/\epsilon)$ becomes more prominent as $\epsilon \to 0$. Therefore in the limit as $\epsilon \to 0$, the difference in power cost between paths obtained with the $\mathcal{PC}$ metric and those obtained with the $\mathcal{KC}$ metric will be negligible.

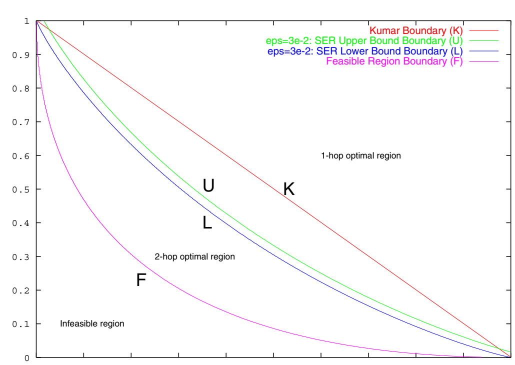

Fig. 2. Boundary of 1-hop v/s 2-hop paths, three points.

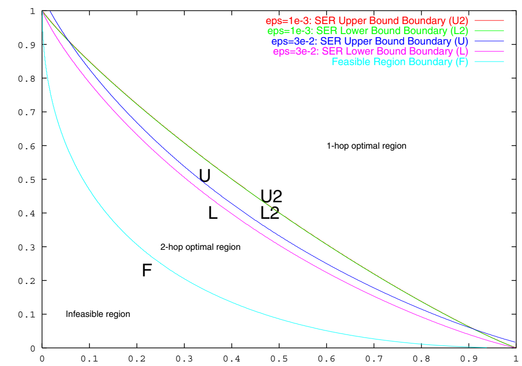

Fig. 3. Boundary of 1-hop v/s 2-hop paths, three points.

D. The Graph of Power Optimal Paths

Given two points, we can determine the path between them that minimizes the power cost. Let $G(\epsilon) = (V, E(\epsilon))$ be the directed graph formed with vertex set V being the set of nodes in the network, and edge $(x, y) \in E(\epsilon)$ just when the edge is part of some power optimal path between two nodes in the network. This is referred to as the graph of power optimal paths [1]. It is clear that if we use the $\mathcal{KC}$ metric, then the graph will not be a function of ϵ.

1) Crossings in Optimal Paths: Power optimal paths computed according to $\mathcal{KC}$ have the property that two power optimal paths will never cross each other [1]. Unfortunately, this property no longer holds when we use the $\mathcal{PC}$ metric. Consider the same triangle as shown in Figure 1, except we let the side AC have distance d_3. From geometry, we know that

$$d_3^2 = d_1^2 + d_2^2 - 2 d_1 d_2 \cos\theta$$

Consider the case $\theta = \pi/2$, $\alpha = 2$, and $d_1 = d_2 = d$. We know that $d_3 = \sqrt{2}d$.

$$\begin{aligned} \mathcal{PC}(\epsilon; (A, C)) &= \frac{2d^2}{a} \log(b/\epsilon) \\ \mathcal{PC}(\epsilon; (A, B, C)) &= \mathcal{PC}(\epsilon; (A, C)) + \frac{2d^2 \log 2}{a} \end{aligned}$$

Therefore, we conclude that in this case at least it is cheaper to send a packet directly from A to C instead of via B. If we pick a fourth point D that is the reflection of B in AC (in the case we are considering, this makes $ABCD$ a square), then by symmetry we know that the power optimal path from B to D is the one-hop path (B, D). Hence, there are two power optimal paths (A, C) and (B, D) that cross each other.

2) $\mathcal{KC}$ optimal and $\mathcal{PC}$ optimal graphs: In the previous section we showed that the graph of $\mathcal{PC}$-optimal paths exhibits crossings. In this section we show an inclusion property relating the graph of $\mathcal{KC}$-optimal paths with the graph of $\mathcal{PC}$-optimal paths. The main result we will establish in this section is that every edge in the graph of $\mathcal{KC}$-optimal paths also occurs in the graph of $\mathcal{PC}$-optimal paths for all values of ϵ.

Lemma 1: A one-hop path that is power optimal by the $\mathcal{KC}$ metric is strictly power optimal by the $\mathcal{PC}$ metric.

We can also see this property in Figure 2, where the $\mathcal{PC}$ metric always picks a one-hop path whenever the $\mathcal{KC}$ metric does.

Theorem 4: Let $G = (V, E)$ be a graph of power optimal paths by the $\mathcal{KC}$ metric, and let $G'(\epsilon) = (V, E'(\epsilon))$ be a graph of power optimal paths by the $\mathcal{PC}(\epsilon; \cdot)$ metric. Then $E \subseteq E'(\epsilon)$.

Given two nodes, there may be multiple paths between them that are optimal in terms of power cost. This implies that the graph of power optimal paths according to either power cost metric need not be unique. As Lemma 1 guarantees strict optimality in terms of the $\mathcal{PC}$ metric, the result from Theorem 4 holds regardless of which graph is chosen for either power cost metric. This also implies that the union of all possible $\mathcal{KC}$ optimal graphs is a subgraph of the graph of $\mathcal{PC}$ optimal paths.

3) Asymptotic Properties of Optimal Paths: As noted in Section IV-C, we expect that the difference in the cost of paths chosen by the $\mathcal{PC}$ metric and $\mathcal{KC}$ metric to be negligible as $\epsilon \to 0$ because the term $\log(b/\epsilon)$ will dominate, reducing the difference between the numerical value of $\mathcal{PC}$ and $\mathcal{KC}$ by equations (13) and (14). In this section we examine the behavior of the optimal paths themselves as $\epsilon \to 0$.

Theorem 5: Let $\mathbf{X}$ be a $\mathcal{PC}(\epsilon; \cdot)$-optimal path and let $\mathbf{X}'$ be a $\mathcal{KC}$-optimal path between points A and B. Then either $\mathbf{X}$ is also $\mathcal{KC}$-optimal, or there exists an $\epsilon' > 0$ such that

$$\mathcal{PC}(\epsilon'; \mathbf{X}') \leq \mathcal{PC}(\epsilon'; \mathbf{X}) \tag{15}$$

Theorem 5 states that for a suitable choice of ϵ, the $\mathcal{PC}(\epsilon; \cdot)$ criterion picks a path that is also optimal by the $\mathcal{KC}$ criterion. Therefore, as $\epsilon \to 0$, the paths chosen by $\mathcal{PC}$ will converge to those chosen by $\mathcal{KC}$.

V. Computing Power Optimal Paths

Computing the optimal paths between points in a network using the $\mathcal{KC}$ metric is a simple task. The reason for this is that the cost of a link between two nodes is simply $a \log(b/\epsilon)$, where a is the attenuation coefficient of the link. Therefore, we can create a cost matrix $\mathbf{M}_{N\times N}$ where entry $m_{i,j}$ the attenuation coefficient of the one-hop path between node i and node j in the network. The power optimal paths are easily obtained by using an all-points shortest-path algorithm on the matrix $\mathbf{M}$ (see e.g., [3]).

Instead, if we solve the end-to-end optimization problem using the $\mathcal{PC}$ metric, the cost of a hop depends on the path that uses the hop by Theorems 2 and 3. Therefore the matrix formulation described above will not compute the correct paths. In this section we describe algorithms that will compute the $\mathcal{PC}$-optimal paths for a graph that are generalizations of standard shortest-path algorithms.

For the generalization, we rely on the existence of two path cost functions $\mathbf{c}$ and $\mathbf{d}$. One of these, $\mathbf{c}$, will correspond to the power cost of the path, and the other will be a suitably defined auxiliary function. We require certain properties of this pair of cost functions.

Property P1. Both cost functions are assumed to satisfy a reversibility criteria, namely the cost of path $(X_0, \ldots, X_n)$ is the same as the cost of path $(X_n, \ldots, X_0)$.

Property P2. Let $\mathbf{X} = (A, X_1, \ldots, X_{n-1}, B)$ and $\mathbf{Y} = (A, Y_1, \ldots, Y_{m-1}, B)$ be two paths between nodes A and B. Further, let $\mathbf{X}' = (A, X_1, \ldots, X_{n-1}, B, Z_1, \ldots, Z_{l-1}, C)$ and $\mathbf{Y}' = (A, Y_1, \ldots, Y_{m-1}, B, Z_1, \ldots, Z_{l-1}, C)$ be the extensions of paths $\mathbf{X}$ and $\mathbf{Y}$ by the same set of hops to node C. The cost functions $\mathbf{c}$ and $\mathbf{d}$ are assumed to satisfy the following property:

$$\begin{aligned} &(\mathbf{c}(\mathbf{X}) < \mathbf{c}(\mathbf{Y})) \quad \wedge \quad (\mathbf{d}(\mathbf{X}) < \mathbf{d}(\mathbf{Y})) \\ \Rightarrow &(\mathbf{c}(\mathbf{X}') < \mathbf{c}(\mathbf{Y}')) \quad \wedge \quad (\mathbf{d}(\mathbf{X}') < \mathbf{d}(\mathbf{Y}')) \end{aligned}$$

In other words, if path $\mathbf{X}$ between two nodes has a lower cost function in terms of both functions $\mathbf{c}$ and $\mathbf{d}$ than path $\mathbf{Y}$, then all extensions of path $\mathbf{X}$ to a third node will have a lower cost (in terms of both $\mathbf{c}$ and $\mathbf{d}$) when compared with extensions to $\mathbf{Y}$. This property allows us to discard path $\mathbf{Y}$ from further consideration since it will never be a subpath of an optimal path between node A and any other node in the network. Note that while this property is stated for the case when both paths are extended by an l-hop path, it can be established by proving it for one-hop extensions and then applying induction. We say that path $\mathbf{X}$ *dominates* path $\mathbf{Y}$. A path $\mathbf{X}$ is said to be *feasible* if it is not dominated by any other path.

Property P3. We assume that a subpath will always have lower cost (using both $\mathbf{c}$ and $\mathbf{d}$ cost functions) than the original path.

Given these cost functions, we now describe algorithms for computing optimal paths in a network. We will instantiate these algorithms using different cost functions to solve the power optimal route computation problem according to the power cost metrics of Theorems 2 and 3. The correctness of our algorithms depends on the following result. We use the notation $\mathbf{p}_{XY}$ to denote a path from X to Y.

Lemma 2: Let $\mathcal{S}_k$ be the set of all feasible paths that have at most k hops. Then every l-hop subpath in $\mathcal{S}_k$ is contained in $\mathcal{S}_l$.

We now establish the following lemma that can be used to construct a set $\mathcal{S}_{l+m}$ given sets $\mathcal{S}_l$ and $\mathcal{S}_m$.

Lemma 3: The following procedure can be used to construct $\mathcal{S}_{l+m}$ given $\mathcal{S}_l$ and $\mathcal{S}_m$:

1) Initialize $\mathcal{S}_{l+m}$ to $\mathcal{S}_l$.
2) For every pair of paths $\mathbf{p}_{AB} \in \mathcal{S}_l$ and $\mathbf{q}_{BC} \in \mathcal{S}_m$
   a) Place the concatenated path $\mathbf{r}_{AC}$ in set $\mathcal{S}_{l+m}$ if it is not dominated by an existing path from A to C in $\mathcal{S}_{l+m}$.
   b) Eliminate all paths from A to C that are in $\mathcal{S}_{l+m}$ that are dominated by $\mathbf{r}_{AC}$.

We use the notation $S_l \otimes S_m$ to denote the operation specified in Lemma 3. Given Lemma 3, the algorithm for computing optimal paths is straightforward. We provide two techniques for computing optimal paths.

Algorithm 1 (Single step): The following algorithm computes $\mathcal{S}_N$ in variable $\mathcal{S}$, where N is the number of nodes in the network.

```
S_1 ← {(i,j): 1 ≤ i,j ≤ N, i ≠ j}
S ← S_1
for i ← 1 to N − 1
    S ← S ⊗ S_1
```

A faster technique for computing $\mathcal{S}_N$ is shown below.

Algorithm 2 (Doubling): The following algorithm computes $\mathcal{S}_N$ in variable $\mathcal{S}$, where N is the number of nodes in the network.

```
S ← {(i, j): 1 ≤ i, j ≤ N, i ≠ j}
for i ← 1 to ⌈lg N⌉
    S ← S ⊗ S
```

Once all the candidate paths are computed, we can pick the optimal path from A to B according to metric $\mathbf{c}$ by picking the least cost path (according to $\mathbf{c}$) among the candidates in $\mathcal{S}_N$. Notice that both algorithms shown above turn into the standard matrix-based algorithms for shortest path computation if $\mathbf{c}$ and $\mathbf{d}$ are the same cost function that is additive.

Finally, we present the cost functions $\mathbf{c}$ and $\mathbf{d}$ for the two power cost metrics derived in Section III. In both cases, $\mathbf{c}(\mathbf{X})$ will always be the power cost $\mathcal{PC}(\epsilon; \mathbf{X})$.

Theorem 6: The following two cost functions satisfy properties $(P1)$–$(P3)$.

$$\mathbf{c}(\mathbf{X}) = \sum_{j=1}^{N} a_j \left(\log(b/\epsilon) + \log(\sum_{i=1}^{N} a_i/a_j) \right)$$

$$\mathbf{d}(\mathbf{X}) = \sum_{j=1}^{N} a_j$$

where a_j is the attenuation coefficient of the jth hop in $\mathbf{X}$.

Theorem 7: The following two cost functions satisfy properties $(P1)$–$(P3)$, assuming that $(b/\epsilon) > 1$.

$$\mathbf{c}(\mathbf{X}) = (b/\epsilon)^{1/m} \left(\sum_{j=1}^{N} a_j^{m/(m+1)} \right)^{(m+1)/m} - \sum_{j=1}^{N} a_j$$

$$\mathbf{d}(\mathbf{X}) = \sum_{j=1}^{N} a_j^{m/(m+1)}$$

where a_j is the attenuation coefficient of the jth hop in path $\mathbf{X}$ and m is the Nakagami m-parameter.

Since the definition of $\mathbf{c}(\mathbf{X})$ in Theorem 6 is the same as the power cost of a path as defined by Theorem 2, we can instantiate Algorithms 1 and 2 using the cost functions from Theorem 6 to compute the power optimal paths when we consider time-invariant attenuation.

Since the definition of $\mathbf{c}(\mathbf{X})$ in Theorem 7 is the same as the power cost of a path as defined by Theorem 3, we can instantiate Algorithms 1 and 2 using the cost functions from Theorem 7 to compute the power optimal paths when we consider large and small-scale fading.

VI. Discussion and Conclusions

This paper describes the additional cost and complexity necessary to guarantee the same error rate across all paths in a multi-hop network and compares the power optimal routes obtained with this criterion to the power optimal routing obtained with a fixed per hop error rate constraint. We examined the power cost of paths under two different models for the symbol error rate of a link, and provided several results that relate the routes obtained with our criterion with those obtained from a fixed per hop constraint. Finally, we provided an algorithm to compute power optimal routes.

There are important issues that this paper does not address: the fixed end-to end error rate constraint does not incorporate a mechanism to prevent congestion at specific nodes. In addition, since power optimal routing does not uniformly distribute traffic, it ends up draining the resources of some nodes more than others. Future investigations will be directed towards evaluating the impact of power optimal routing on the network lifetime [9], [10].

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